A Hybrid PEE-FDTD Algorithm for Accelerated Time Domain Analysis of Electromagnetic Waves in Shielded Structures

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***Abstract*— A new algorithm for the time domain analysis of electromagnetic waves in shielded structures is presented. The algorithm combines the FDTD with a recently developed partial eigenfunction expansion (PEE) scheme to obtain acceleration in numerical calculation and savings in computer memory. An example of the application of the algorithm is presented showing an overall speed improvement.**

I. Introduction

TIME DOMAIN techniques are gaining increased popularity in the analysis of electromagnetic waves because of their ability to treat complicated geometries over a wide frequency range. Because the most popular time domain algorithms are explicit, the stability of the algorithm puts a restriction on the maximum allowable time step. Additionally, to obtain fine resolution of fields near singularities, mesh size is reduced thereby increasing the computer storage. The computation time and memory requirements are, therefore, critical parameters for time domain algorithms. Because of this, some research effort has recently been devoted to the acceleration of the traditional methods by the application of the signal processing [2] techniques, graded mesh schemes [1] or elimination of redundant field components [3].

This contribution presents a new algorithm for shielded structures that consists of the replacement of the FDTD calculation in homogeneous shielded subregions by eigenfunction expansion. The eigenfunction expansion schemes in time domain proposed in [4] rely on the expansion of the unknown functions of selected space variables into series of basis functions and the application of method of moment procedure to find the expansion coefficients. One version of the algorithm, called the Partial Eigenfunction Expansion (PEE), is obtained if the function expansion is done with respect to two selected space coordinates while the third spatial coordinate and time are discretized in a way analogical to the finite difference scheme. By combining such an approach with a classical FDTD algorithm, an improvement in speed can be obtained for an important class of shielded structures.

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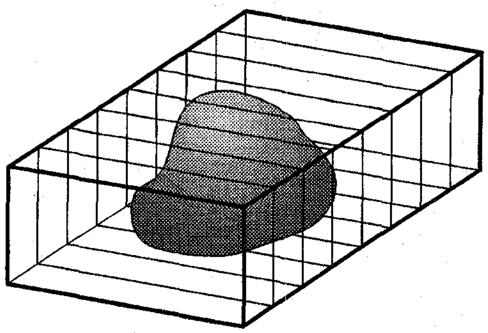

Fig. 1 A structure discretized along one coordinate.

II. Formulation

As the FDTD is a well known technique, let us begin with the presentation of the second scheme, namely the Partial Eigenfunction Expansion (PEE). Consider a dielectric inhomogeneity located in a rectangular waveguide (Fig. 1). In the PEE, the computational space is sliced into subdomains and the fields are expanded on each subdomain (slice) into series of expansion functions that depend only on transverse coordinates and fulfill the boundary conditions on the guide periphery. The function expansion is done in two dimensions for each slice separately, while the variations in the third spatial dimension and time are handled using the finite difference approximations. Suppose the structure was divided into K slices in the z direction and the slices are uniformly spaced by the distance Δd. Using the finite difference approximation of derivatives in the z directions, the Maxwell's equations can be written in the following operator form

$$\frac{\partial}{\partial t} f_k = \mathbf{L}_{1t}^k g_k + \mathbf{L}_{1z}^k (g_{k+1} - g_k)$$
$$\frac{\partial}{\partial t} g_k = \mathbf{L}_{2t}^k f_k + \mathbf{L}_{2z}^k (f_k - f_{k-1}) \tag{1}$$

where f_k and g_k are vector functions representing the electric and magnetic field at the k-th slice and

$$\mathbf{L}_{1t}^k = \frac{1}{\epsilon_0 \epsilon_r^k(x,y)} \nabla_t \times (\cdot) \qquad \mathbf{L}_{1z}^k = \frac{1}{\epsilon_0 \epsilon_r^k(x,y)\Delta d} \hat{z} \times (\cdot)$$
$$\mathbf{L}_{2t}^k = \frac{-1}{\mu_0 \mu_r^k(x,y)} \nabla_t \times (\cdot) \qquad \mathbf{L}_{2z}^k = \frac{-1}{\mu_0 \mu_r^k(x,y)\Delta d} \hat{z} \times (\cdot) \tag{2}$$

In the above equations, we have denoted a unit vector in the z direction by $\hat{z}$.

The fields are now expanded separately on each slice according to

$$f_k(t) = \sum_i a_{i_k}(t) f_{i_k}(x,y)$$
$$g_k = \sum_i b_{i_k}(t) g_{i_k}(x,y)$$
$$k = 1 \dots K \tag{3}$$

where a_{i_k}, b_{i_k} are the expansion coefficients (time dependent) and $f_{i_k}(x,y), g_{i_k}(x,y)$ are the basis functions. The next step is to introduce the expansion (5) into (3) and replace time derivatives by finite difference formulas. This results in equations in which the only unknowns, for a fixed time instant, are the expansion coefficients at all slices of the structure. To evaluate the expansion coefficients we take the inner product of equations valid for a given slice with expansion functions and use the orthogonality property of the expansion functions. As a result, we arrive at [4]:

$$\underline{a}^{\,n} = \underline{a}^{\,n-1} + \Delta t \underline{\underline{A}}\,\underline{b}^{\,n-1/2}$$
$$\underline{b}^{\,n+1/2} = \underline{b}^{\,n-1/2} + \Delta t \underline{\underline{B}}\,\underline{a}^{\,n} \tag{4}$$

where Δt is the time step, $\underline{a}$ and $\underline{b}$ are column vectors containing expansion coefficients for all slices and superscript n denotes the time step. The matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ contain the inner products and have the following structure

$$\underline{\underline{A}} = \text{qdiag}[\underline{\underline{A'}}^{\,k}, \underline{\underline{A''}}^{\,k}] \qquad \underline{\underline{B}} = \text{qdiag}[\underline{\underline{B'}}^{\,k}, \underline{\underline{B''}}^{\,k}] \tag{5}$$

The elements of the submatrices are given

$$A'^{\,k}_{ij} = \langle (\mathbf{L}^k_{1t} - \mathbf{L}^k_{1z}) g_{j_k}, f_{i_k} \rangle \qquad A''^{\,k}_{ij} = \langle \mathbf{L}^k_{1z} g_{j_{k+1}}, f_{i_k} \rangle$$
$$B''^{\,k}_{ij} = \langle (\mathbf{L}^k_{2t} + \mathbf{L}^k_{2z}) f_{j_k}, g_{i_k} \rangle \qquad B'^{\,k}_{ij} = - \langle \mathbf{L}^k_{2z} f_{j_{k-1}}, g_{i_k} \rangle \tag{6}$$

Equation (6) shows that expansion coefficients for all slices are updated at each time step as a result of mutual interactions of fields due to the inhomogeneity introduced by space dependence of constitutive parameters. As far as numerical cost is concerned, the most critical point in the PEE algorithm is the calculation of the inner products on inhomogeneous slices. However, for inhomogeneous slices a classical FDTD algorithm can be used. Combining these two time domain techniques, we create a hybrid method in which different algorithms are used in different parts of the computational space. The FDTD is used in the regions in which a fine resolution of field is necessary (eg. near edges, media interfaces), and the PEE is applied in the homogeneous subregions. This hybrid approach results in savings in numerical effort and computer memory. This is because the PEE is extremely efficient for homogeneous slices as matrices $\underline{\underline{A'}}^{\,k}, \underline{\underline{A''}}^{\,k}$ and $\underline{\underline{B'}}^{\,k}, \underline{\underline{B''}}^{\,k}$ are diagonal, so that the computations are fast, especially when the expansion functions are chosen in such a way that they constitute a set of eigenfunctions of the Laplace operator defined on 2D region forming a slice. In that case, each expansion function satisfies the boundary condition and field equations globally over entire slice. As a result, very few expansion terms are needed to accurately describe field at each slice. The FDTD and PEE algorithms are interfaced at a common slice. The transition form the FDTD to PEE is done in the following way. Given a field distribution at the $z = z_k$, provided by the FDTD part of the algorithm, the expansion coefficients at this slice are found by taking the inner product with each basis functions of the PEE. To switch from PEE to FDTD the series (3) are calculated at the interface plane at the points required by FDTD.

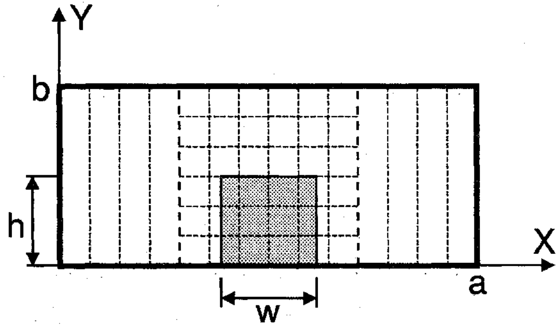

Fig. 2 Geometry of the guide used in the numerical test showing FDTD and PEE meshes. (Slab centered with respect to $x = a/2$, $\epsilon_r = 2.5$; dimensions: $h = 4$ mm, $w = 4$ mm, $a = 20$ mm, $b = 6$ mm.)

TABLE I
COMPARISON OF THE RESULTS AND CPU TIMES (DELL 486/66) FOR THE CUTOFF FREQUENCIES OF EH_{11} MODE IN THE 20 BY 6 mm RECTANGULAR GUIDE LOADED WITH A DIELECTRIC SLAB. $\epsilon_r = 2.5$, $w = 4$mm, $h = 4$ mm USING A HYBRID ALGORITHM WITH NUMBER OF EXPANSION FUNCTIONS N AND THE LOCALIZATION OF THE INTERFACE PLANES AS PARAMETERS

N	PEE-FDTD	Error rel. to FDTD	CPU FDTD part	CPU PEE part	CPU combined	Speed up rel. to FDTD (53s)
Interface of algorithms at $x = 7.5, x = 12.5$ mm						
1	20.2075 GHz	+ 0.1%	14s	4s	18s	2.9
3	20.1975 GHz	+0.05%	14s	9s	23s	2.3
5	20.1975 GHz	+0.05%	14s	13s	27s	1.96
Interface of algorithms at $x = 5, x = 15$ mm						
1	20.1875 GHz	0%	28s	3s	31s	1.7
3	20.1875 GHz	0%	28s	6s	34s	1.55

III. NUMERICAL EXAMPLE

In order to verify the hybrid algorithm, the cutoff frequency of the EH_{11} of a rectangular guide loaded with a dielectric slab shown in Fig. 2 was computed and compared with the results obtained with a classical FDTD technique. For both algorithms the identical excitation, space discretization ($\Delta d =$ .5 mm), time step, and number of samples were assumed. The results are given in Table I. For the FDTD algorithm alone, the CPU time for the assumed discretization mesh was 53 s. For the hybrid algorithm, the slab region was treated with the FDTD and the lateral homogeneous regions were calculated with PEE. The CPU time depends on the localization of the interface plane and the number of expansion functions used in the PEE part of the algorithm. When the interface is at $x_1 =$ 7.5 and x_2 =12.5 mm, ie. when the FDTD mesh is terminated only one slice away from the inhomogeneous region, the CPU time of the hybrid algorithm varies from 18 s (for one expansion function) to 27 s (for five expansion functions), with most of the time (14 s) consumed by the FDTD computations. The error introduced by low number of expansion function is the largest if only one term is used, but for this structure it is less than 0.1% compared the result obtained from pure FDTD calculations. The error decreases as the interface plane is moved away from the inhomogeneity. This is due to the fact that higher-order terms in the field expansion correspond to higher-order waves traveling in the lateral direction. If the interface planes are located at $x_1 = 5$ and $x_2 = 15$ mm, only one term in the PEE part is sufficient to obtain exactly the same results as with purely FDTD technique. The CPU time for this case is 31 s, of which 28 s is spent in the FDTD part.

IV. Conclusions

A new hybrid PEE-FDTD algorithm for the time domain analysis of electromagnetic waves in shielded structures was introduced. The obtained results indicate that it is possible to obtain the acceleration of time domain calculation by using the PEE algorithm in homogeneous parts of the structure. The results presented in this letter indicate that it is possible to obtain improvement in the speed of time domain computation of 3D and 2D shielded structures in which the homogeneous regions are predominant, such as microstrip lines, coplanar guides, and discontinuities in planar guides.

References

[1] S. Xiao, R. Valdieck, and H. Jin, "A fast two dimensional FDTD full-wave analyser with adaptive mesh size," *IEEE MTT-S Int. Symp. Dig.*, vol. 2, pp. 783–786, June 1992.

[2] W. Ko and R. Mittra, "A combination of FD-TD and Prony methods for analyzing microwave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 2176–2181, Dec. 1991.

[3] M. Okoniewski, "Vector wave equation 2-D–FDTD method for guided wave problems," *IEEE Microwave and Guided Wave Lett.*, vol. 3, pp. 307–309, Sept. 1993.

[4] M. Mrozowski, "Function expansion algorithms for the time domain analysis of shielded structures supporting electromagnetic waves," *J. Numerical Modelling*, vol. 7, pp. 77–84, Mar. 1994.